2013

MATHEMATICS-III

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

1. Answer any *ten* from the following: 10 x 2 = 20

i) If $f(x) = x\sin x, -\pi \le x \le \pi$ be presented in Fourier series as $a_0/2 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then find the value of a_0.

ii) Show that every function can be expressed as a sum of even and odd functions.

iii) If $f(x)$ is an odd function, then find the Fourier transform of $f(x)$.

iv) If $\mathcal{F}\{f(x)\} = F(s)$, then show that $\mathcal{F}\{e^{iax}f(x)\} = F(s+a)$, when $\mathcal{F}$ stands for Fourier transform.

v) Find the value of m such that $3y - 5x^2 + my^2$ is a harmonic function.

vi) Determine the poles of the function $f(z) = z^2/[(z-1)^2(z+2)]$

vii) Find the residue of $\frac{z^2}{z^2-3^2}$ at $z = 3i$.

viii) Evaluate $\oint_C \frac{z}{z^2-1}\, dz$ where C: |z|=2.

ix) A box contains 6 white and 4 black balls. One ball is drawn at random. What is the probability that the ball drawn is white?

x) If A and B are any two events such that $P(A \cap B) = \frac{1}{2}$, $P(A^c \cap B^c) = \frac{1}{3}$, $P(A) = P(B) = p$, then find the value of p.

xi) If a Poisson variate is such that $P(X = 1) = P(X = 2)$, then find $P(X = 4)$.

xii) Find the singular point of

$$x(x-1)^2(x+2)\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + (x^3+1)y = 0.$$

xiii) What is Bessel equation? What is Bessel function of first kind? Write down the formula.

xiv) Evaluate $J_{\frac{1}{2}}(x)$.

xv) What is one-dimensional heat equation? Also write down the formula for Wave equation.

GROUP – B

Answer any *five* questions taking at least *one* question from each Module. 5 × 10 = 50

Module-I

2. a) Find the Fourier series to represent the function:

$$f(x) = -x,\ -2 < x \le 0$$
$$= x,\ 0 \le x \le 2$$ 5

b) Find the Fourier sine and cosine transforms of the function:

$$f(x) = 1;\ 0 \le x < a$$
$$= 0;\ x \le a$$ $2\frac{1}{2} + 2\frac{1}{2}$

3. a) State Parseval's Identity on Fourier. Find the Fourier transform of $f(x) = \begin{cases} 1; |x| \le a \\ 0; |x| > a \end{cases}$. Hence using Parseval's Identity find the value of $\int_0^\infty \frac{\sin^2 ax}{x^2} dx$. 1+3+2

b) If $\mathcal{F}\{f(x)\} = F(S)$, then show that

$\mathcal{F}\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$. Use it to find $\mathcal{F}\left\{e^{-\frac{9}{2}x^2}\right\}$, given that

$\mathcal{F}\left\{e^{-\frac{x^2}{2}}\right\} = \sqrt{2\pi}\, e^{-\frac{s^2}{2}}$, where $\mathcal{F}$ stands for Fourier transform. 2+2

Module-II

4. a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy-Rieman equations are satisfied at that point. 5

b) If $f(z)$ is analytic, prove that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2.$ 5

5. a) State Cauchy Integral formula and use it to evaluate

$\int \frac{\text{Cos}(\pi z)}{z^2-1} dz$, around a rectangle with vertices $2 \pm i$,

$-2 \pm i$. 5

b) Use residue calculus to evaluate the integral
$\int_0^{2\pi} \frac{1}{5-4\sin\theta} d\theta.$ 5

Module-III

6. a) A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$ +K

Obtain the value of K and estimate P(X<6) and P(0<X<5). 5

b) In a bolt factory, the machines M_1 , M_2 and M_3 manufacture respectively 25%, 35% and 40% of the total product. Of their outputs 5%, 4% and 2% respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine M_3 ? 5

7. a) A car hire firm has two cars, which it hires out day by day. The No. of demands for a car on each day is distributed as a Poisson distribute with mean 1.5. Calculate the proportion of days on which (i) neither car is used, (ii) some demands are refused. 5

b) The length of the life of tyre manufactured by a company follows a continuous distribution given by the density function

$f(x) = k/x^3$, $1000 \leq x < 1500$

$= 0$, elsewhere

Find k and find the probability that a randomly selected tyre would function for at least 1200 hours. 5

Module-IV

8. Solve the one-dimensional heat flow equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\ x \geq 0, t \geq 0$$

Subject to the conditions:

i) $u = 0$, when $x = 0, t > 0$

ii) u = 1, when $0 < x < 1$, t = 0

= 0, when $x \geq 1$, t = 0

iii) $u(x,t)$ is bounded. 10

9. a) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. 6

b) When n is a positive integer, show that

$J_{-n}(x) = (-1)^n \ J_n(x)$. 4

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