2013

NUMERICAL METHODS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Question)

1. Choose the correct alternatives for any *ten* of the following: 10 x 1 = 10

i) The ratio of absolute error of the value is

a) Relative error b) Absolute error

c) Truncation error d) Inherent error.

ii) The significant digit of 0.0001234 is

a) 7 b) 4

c) 8 d) 6.

iii) The percentage error in approximation of 4/3 to 1.3333 is

a) 0.0025% b) 25%

c) 0.000025% d) 0.25%

iv) If the interval of differencing is unity and $f(x) = ax^2$ (a is constant), which one of the following choices is wrong?

a) $\Delta f(x) = a(2x + 1)$

b) $\Delta^2 f(x) = 2a$

c) $\Delta^3 f(x) = 2$

d) $\Delta^4 f(x) = 0$.

v) In Simpson's $1/3$ rule of finding $\int_a^b f(x)dx$, $f(x)$ is approximated by

a) line segment b) parabola

c) circular sector d) part of ellipse.

vi) Runge-Kutta formula has a truncation error which isof the order of

a) h^2 b) h^3

c) h^4 d) h^5

vii) If $f(x) = \frac{1}{x^2}$, then the divided difference *f(a, b)* is

a) $\frac{a+b}{(ab)^2}$ b) $\frac{-a-b}{(ab)^2}$

c) $\frac{1}{a^2-b^2}$ d) $\frac{1}{a^2} - \frac{1}{b^2}$.

viii) The method of Iteration formula $\varphi(x)$ must satisfy

a) $|\phi'(x)| < 1$ b) $|\phi'(x)| > 1$

c) $|\phi'(x)| = 1$ d) $|\phi'(x)| = 2$.

ix) Which of the following methods if an iterative method?

a) Gauss-elimination method

b) Gauss-Seidel method

c) LU-factorization method

d) Matrix-inversion method.

x) Regula-falsi method is

a) conditionally convergent

b) linearly convergent

c) divergent

d) none of these.

xi) Simpson's one-third rule is applicable only when the number of sub-intervals is

a) even b) odd

c) both even & odd d) none of these.

xii) In LU-factorization method, the given system equation represented by AX=B is converted to another system LUX=B where U is

a) lower triangular matrix

b) upper triangular matrix

c) identity matrix

d) null matrix.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 x 5 = 15

2. Show that if Δ operates on n, then $\Delta\binom{n}{x+1} = \binom{n}{x}$ and hence $\sum_{n=1}^{N}\binom{n}{x} = \binom{n+1}{x+1} - \binom{1}{x+1}$.

3. Evaluate $\int_0^1 \cos x\, dx$, taking five equal intervals. Explain the reason behind your choice of integration formula used.

4. Apply Lagrange's interpolation formula to find $f(x)$ using following table:

x:	1	2	3	4	7
$f(x)$:	2	4	8	16	128

5. Solve by using Euler's method the following differential equation for $x = 1$ by taking $h = 0.2$, $\frac{dy}{dx} = xy$, $y = 1$ when $x = 0$.

6. Solve the system of linear equations by Gauss-Jordan method:

$2x + y + z = 0$

$3x + 2y + 3z = 0$

$x + 4y + 9z = 16$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 x 15 = 45

7. a) Compute $f(0.23)$ and $f(0.29)$ using suitable formula from the table given below: 7

x:	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

b) Describe geometric significance of Simpson's $\frac{1}{3}$ rule. 5

c) Determine the absolute error E_A of the following approximate number given their relative error

$x_A = 67.84,\ E_R = 1\%$. 3

8. a) Using Gauss-Seidel method find the solution of the following system of linear equations correct up to two decimal places:

$3x + y + 5z = 13, 5x - 2y + z = 4, x + 6y - 2z = -1.$ 7

b) Solve the equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$, for $y(0.1)$ and $y(0.2)$, using Runge-Kutta method of the fourth order. 8

9. a) Round-off 35.7218 to four significant figures 1

b) What is interpolation? Prove that

$$f(x) \simeq y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n y_0.$$

1+5

c) Prove that $\nabla^r y_k = \nabla^r y_{k+r}$. 3

d) Find the missing term: 5

x:	1	2	3	4	5	6	7
$f(x)$:	2	4	8	?	32	64	128

10. a) Prove the convergence of Newton-Raphson method. Hence find the cube root of 10 up to 5 significant figures by Newton-Raphson mmethod. 5+5

b) Evaluate $\int_0^{0.6} \frac{dx}{\sqrt{1-x^2}}$, using Weddle's rule taking 12 equal sub-intervals. 5

11. a) Find the polynomial $f(x)$ and hence calculate $f(5.5)$ for the given data:

x:	0	2	3	5	7
$f(x)$:	1	47	97	251	477

b) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y(1) = 1$. Evaluate $y(1.2)$ by modified Euler's method correct up to 4 decimal places. 6

c) Solve the following system of equations by L-U decomposition method: 5

$x + y - z = 2, 2x + 3y + 5z = -3, 3x + 2y - 3z = 6.$

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