2013

FIELD THEORY

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Question)

1. Choose the correct alternatives for any *ten* of the following: 10 x 1 = 10

i) If a vector field $A = x\hat{a}x + y\hat{a}y + k\hat{a}z$ represents a magnetic field, the value of k is

a) 1 b) 2

c) -1 d) -2.

ii) The capacitance of an isolated spherical conductor of diameter 1 cm is

a) 0.556 pF b) 0.95 μF

c) 0.556 μF d) 0.95 pF.

iii) The work involved in moving a charge of 1 Coulomb from (6, 8, -10) to (3, 4, -5) along a straight line in the field $E = -x\hat{a}x + y\hat{a}y - 3\hat{a}z$ is

a) 25.5 J b) 15 J

c) 20 J d) None of these.

iv) A scalar potential $\phi = xyz$, then the vector $F = Grad\ \phi$ is

a) irrotational b) solenoidal

c) both (a) & (b) d) none of these.

v) The direction of force on a conductor carrying current in positive y-axis and placed in magnetic field directed in positive x-axis will be

a) negative x-axis b) negative y-axis

c) positive z-axis d) negative z-axis

vi) The magnetic field intensity (in A/m) at the centre of a circular coil of 1 m diameter and carrying a current of 2A is

a) 8 b) 4

c) 3 d) 2.

vii) The value of V (1/r) is, where $\bar{r} = x\hat{a}x + y\hat{a}y + z\hat{a}z$

a) $\bar{r}/r^2$ b) $-\bar{r}/r^3$

c) $-\bar{r}/r^2$ d) none of these

viii) For the volume charge density ρ, the divergence of the electric field intensity will be

a) $\epsilon_0\ \rho$ b) $\epsilon_0^2\ \rho$

c) $\frac{\rho}{\epsilon_0^2}$ d) $\frac{\rho}{\epsilon_0}$

ix) A transmission line is said to be distortionless if

a) $\frac{R}{G} = \frac{C}{L}$ b) $\frac{R}{G} = \frac{L}{C}$

c) RG = LC d) R = 0.

x) Which of the following is zero?

a) Grad Div $\bar{A}$ b) Div Grad $\bar{A}$

c) Div Curl $\bar{A}$ d) Curl Curl $\bar{A}$.

xi) The intrinsic impedance of free space is given by

a) 333 ohm b) 377 ohm

c) 4π ohm d) 2π ohm.

xii) For a lossless transmission line the characteristic impedance is given by

a) sqrt (C/L) b) sqrt (L/C)

c) 2π * sqrt (C/L) d) 2π * sqrt (L/C).

xiii) Which of the following is a mathematically incorrect expression

a) grad div b) curl grad

c) div grad d) curl curl.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 x 5 = 15

2. The vector potential $\vec{A}$ and the scalar potential φ in a certain region of space are given by

$$\vec{A} = \frac{1}{2}\alpha t(\vec{a}yx - \vec{a}xy)$$

$$\varphi = \frac{1}{4}\alpha(x^2 + y^2)$$

Where α is a constant. Calculate the electric and magnetic fields.

3. State and explain Stoke's theorem.

4. a) What do you mean by skin effect? 2

b) If the skin depth is 80 μm at 4 MHz in a certain conducting medium, calculate the skin depth if the frequency is changed to 16 MHz. 3

5. A transmission line has characteristic impedance of 70 Ω and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and capacitance per meter of the line.

6. Use Gauss law to find the electric field at a point both (i) inside (ii) outside a uniformly charged sphere of radius *a*.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 x 15 = 45

7. a) What is Lorentz gauge? 2

   b) Use this gauge to obtain the inhomogeneous wave equations for the scalar and vector potentials. 10

   c) Indicate how solutions of the above wave equations lead to retarded scalar and vector potentials. 3

8. a) Obtain Poyntin theorem for conservation of energy in electromagnetic fields & discuss the physical meaning of each term in the resulting equation. 6+2

   b) An EM wave travels in free space with electric field component

   $E = \left(10a_y + 5a_z\right)\cos(wt + 2y - 4z)\ V/m$

   Determine the following:

   i) ω & λ

   ii) the magnetic field component

   iii) the time average power in the wave. 2+2+3

9. a) State Biot-Savart's law. 2

   b) An infinitely long wire is carrying a current I. Find the magnetic field intensity due to this current at a point which is r m away from the wire. 4

   c) Find out the inductance of a long solenoid of radius r and N no. of turns. 4

   d) A square coil 3 m × 3 m is allowed to fall freely along a vertical keeping two opposite sides vertical, from the top of a tower 80 m high. If the magnetic field perpendicular to the plane of the coil is given by
   $B(y) = [(5y/1000) + 0.0005]\ \ wb/m^2$

Find the induced emf in the coil just before hitting the ground. Here, y is the vertical position coordinate measured from the tower top. Acceleration due to gravity is 9.81 m/s^2. 5

10. a) Derive the Maxwell's equation in electrostatic field. 6

    b) Why electrostatic field is called conservative field? 4

    c) Derive Poisson's and Laplace's equation. 5

11. Write short notes on any *three* of the following: 3 x 5

    a) Magnetic material

    b) Propagation constant

    c) Modified Ampere's circular law

    d) Inductor energy and energy density.

==========