CS/BCA/SEM-1/BM-101/2013-14

2013

MATHEMATICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Question)

1. Choose the correct alternatives for any *ten* of the following: 10 x 1 = 10

i) If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

a) $\Delta > 0$ b) $\Delta < 0$

c) $\Delta = 0$ d) none of these.

ii) If the matrix $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is singular then the value of λ is

a) 0 b) 4

c) 2 d) -1

iii) If A be a matrix whose inverse exists then which of the following is not true?

a) $(A^T)^{-1} = (A^{-1})^T$ b) $A^{-1} = (\det A)^{-1}$

c) $(A^2)^{-1} = (A^{-1})^2$ d) None of these.

iv) $\frac{\partial}{\partial x}(e^{xy}) =$

a) e^{xy} b) xe^{xy}

c) ye^{xy} d) none of these.

v) The degree of the function $f(x,y) = \tan^{-1}\frac{y}{x}$ is

a) 1 b) 0

c) 2 d) none of these.

vi) The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ is

a) $\frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$

c) $\frac{1}{3}\begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$ d) none of these.

vii) The value of $\int \frac{dx}{x \log x}$ is

a) $\log|x| + c$ b) $\log\ |\log x| + c$

c) $x \log|x| + c$ d) none of these.

viii) If α, β and γ be the roots of the equation $x^3 + 7x - 2 = 0$ then $\sum \alpha^2$

a) 0 b) 14

c) -14 d) 4

ix) Which of the following is a null set?

a) $A = \{0\}$

b) $A = \{\phi\}$

c) $A = \{x: x \text{ is an integer } \& \ 1<x<2\}$

d) none of these.

x) The value of $\text{Lim}_{x\to 0}\frac{\sin x}{x}$ (where x is radian) is

a) 1 b) 0

c) ∞ d) -1

xi) The conic $\frac{l}{r} = 1 - e\cos\theta$ represents a parabola if

a) $e = 1$ b) $e > 1$

c) $e < 1$ d) none of these.

xii) What is the value of the following limit?

$$\text{Lim}_{x\to 0}(1+x)^{1/x}$$

a) 1 b) e

c) 0 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 x 5 = 15

2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}\,dx$.

3. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

4. If $u = \tan^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ two of whose roots are in the ratio 3:2.

6. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 x 15 = 45

7. a) State Decartes' rule of sign. Using this rule find the nature of the root of the equation
$x^4 - 7x^3 + 21x^2 - 9x + 21 = 0.$ 5

b) Solve the following system of linear equations by Cramer's rule: 5

$x - y + 2z = 1, \quad x + y + z = 2, \quad 2x - y + z = 5.$

c) If by a transformation of rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$. 5

8. a) If G be a group such that $(ab)^2 = a^2b^2 \forall\, a, b \in G$, show that the group G is Abelion. 5

b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8}\log 2.$ 5

c) If $y = e^{-x}\sin x$, then show that $y_4 + 4y = 0.$ 5

9. a) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal and hence find A^{-1}. 4

b) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ then show that $A^2 - 2A + I_2 = O_2$.
Hence obtain $A^{-1}$and also find A^{100}.

c) Reduce the following equation to the canonical form and determine the nature of the conic represented by it:
$8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0.$

10. a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. 6

b) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$. 4

c) If α, β, γ are the three roots of $x^3 + px^2 + qx + r = 0$, obtain the value of $\sum(\alpha - \beta)^2$. 5

11. a) State Rolle's theorem. Examine whether Rolle's theorem is applicable or not for the function $f(x) = 1 - |x - 1|, \forall x \in [0, 2]$.

b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

c) Find for what values of x, the following expression is maximum and minimum respectively:
$2x^3 - 21x^2 + 36x - 20$ 5+5+5

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